11 Marking scheme: Worksheet

1 a $Q = VC = 9.0 \times 30 \times 10^{-6}$ [1]

$Q = 2.7 \times 10^{-4}$ C (270 μC) [1]

b Number of excess electrons $= \frac{Q}{e} = \frac{2.7 \times 10^{-4}}{1.6 \times 10^{-19}}$ [1]

number $= 1.69 \times 10^{15} \approx 1.7 \times 10^{15}$ [1]

2 a The charge is directly proportional to the voltage across the capacitor. Hence doubling the voltage will double the charge. [1]

charge = 2 × 150 = 300 nC [1]

b Since $Q \propto V$ for a given capacitor, increasing the voltage by a factor of three will increase the charge by the same factor. [1]

charge = 3 × 150 = 450 nC [1]

3 a $E = \frac{1}{2}V^2C = \frac{1}{2} \times 9.0^2 \times 1000 \times 10^{-6}$ [1]

$E = 4.05 \times 10^{-2}$ J $\approx 4.1 \times 10^{-2}$ J [1]

b For a given capacitor, energy stored $\propto$ voltage^2. [1]

energy $= 2^2 \times 4.05 \times 10^{-2} \approx 0.16$ J [1]

4 a $C_{total} = C_1 + C_2$ [1]

$C_{total} = 20 + 40 = 60$ nF [1]

b $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$ [1]

$\frac{1}{C_{total}} = \frac{1}{100} + \frac{1}{500} = 0.012\ \mu F^{-1}$ [1]

$C_{total} = \frac{1}{0.012} \approx 83\ \mu F$ [1]

c $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ [1]

$\frac{1}{C_{total}} = \frac{1}{10} + \frac{1}{50} + \frac{1}{100} = 0.13\ \mu F^{-1}$ [1]

$C_{total} = \frac{1}{0.13} \approx 7.7\ \mu F$ [1]

d Total capacitance of the two capacitors in parallel = 50 + 50 = 100 μF. [1]

$\frac{1}{C_{total}} = \frac{1}{50} + \frac{1}{100} = 0.03\ \mu F^{-1}$ [1]

$C_{total} = \frac{1}{0.03} \approx 33\ \mu F$ [1]

e Total capacitance of the two capacitors in series is 83 μF (from **b**). [2]

$C_{total} = 83 + 50 = 133\ \mu F \approx 130\ \mu F$ [1]

5 a $C_{total} = C_1 + C_2$ [1]

$C_{total} = 100 + 500 = 600\ \mu F$ [1]

b The potential difference across parallel components is the same and equal to 1.5 V. [1]

c $Q = VC = 1.5 \times 600 \times 10^{-6}$ [1]

$Q = 9.0 \times 10^{-4}$ C (900 μC) [1]

d $E = \frac{1}{2}QV = \frac{1}{2} \times 9.0 \times 10^{-4} \times 1.5$ [1]

$E = 6.75 \times 10^{-4}\ J \approx 6.8 \times 10^{-4}\ J$ [1]

6 a $E = \frac{1}{2}V^2C = \frac{1}{2} \times 32^2 \times 10\ 000 \times 10^{-6}$ [1]

$E = 5.12\ J \approx 5.1\ J$ [1]

b $P = \frac{E}{t} = \frac{5.12}{0.300}$ [1]

$P \approx 17$ W [1]

7 a $Q = VC = 12 \times 1000 \times 10^{-6}$ [1]

$Q = 1.2 \times 10^{-2}$ C (12 mC) [1]

b i $C_{total} = C_1 + C_2$ [1]

$C_{total} = 1000 + 500 = 1500\ \mu F$ [1]

ii $V = \frac{Q}{C}$ (The charge Q is conserved and C is the total capacitance.) [1]

$V = \frac{1.2 \times 10^{-2}}{1500 \times 10^{-6}} = 8.0$ V [1]

8 a $I = \frac{V}{R} = \frac{6.0}{100 \times 10^3}$ [1]

$I = 6.0 \times 10^{-5}$ A (60 μA) [1]

b After a time equal to one time constant, τ, the current will be e^{-1} (37%) of its initial value.

The voltage after time τ will be:

$0.37 \times 6.0 \approx 2.2$ V [1]

Therefore $\tau \approx 15$ s (allow ±2 s) [1]

c $\tau = CR$ [1]

$C = \frac{\tau}{R} = \frac{15}{100 \times 10^3} = 1.5 \times 10^{-4}$ F (150 μF) [1]

9 a $\tau = CR = 220 \times 10^{-6} \times 1.2 \times 10^6$ [1]

$\tau = 264\ s \approx 260\ s$ [1]

b i $I = \frac{V}{R} = \frac{8.0}{1.2 \times 10^6}$ [1]

$I = 6.67 \times 10^{-6}\ A \approx 6.7\ \mu A$ [1]

ii After a time equal to two time constants, the current will be:

$e^{-2} \times 6.67 \times 10^{-6} = (0.37)^2 \times 6.67 \times 10^{-6}$ [1]

current $\approx 9.1 \times 10^{-7}$ A (0.91 μA) [1]

iii $V = V_0\ e^{-t/CR}$ [1]

$V = 8.0 \times e^{-(50/264)}$ [1]

$V = 6.62\ V \approx 6.6\ V$ [1]

10 $CR = 100 \times 10^{-6} \times 470 \times 10^{3}$ [1]
$CR = 47$ s [1]
$V = V_0 \, e^{-t/CR}$ [1]
$\frac{V}{V_0} = 0.5$, therefore $0.5 = e^{-t/47}$ [1]
$\ln(0.5) = -\frac{t}{47}$ so $t = -\ln(0.5) \times 47 = 32.6$ s ≈ 33 s [1]

11 The capacitors are in *parallel*, so the total capacitance = $3C$. [1]
The total charge Q remains constant. [1]
The energy stored by a capacitor is given by $E = \frac{Q^2}{2C}$. [1]
$E_{\text{initial}} = \frac{Q^2}{2C}$ and $E_{\text{final}} = \frac{Q^2}{2(3C)}$ [1]
Fraction of energy stored = $\frac{E_{\text{final}}}{E_{\text{initial}}} = \frac{Q^2/2(3C)}{Q^2/2C} = \frac{1}{3}$ [1]
Fraction of energy 'lost' as heat in resistor = $1 - \frac{1}{3} = \frac{2}{3}$. [1]
The resistance governs how long it takes for the capacitor to discharge. The final voltage across each capacitor is independent of the resistance. Hence, the energy lost as heat is independent of the actual resistance of the resistor. [1]